

Dressed States of a two component Bose-Einstein Condensate.

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A condensate with two internal states coupled by external electromagnetic radiation, is described by coupled Gross Pitaevskii equations, whose eigenstates are analogous to the dressed states of quantum optics. We solve for these eigenstates numerically in the case of one spatial dimension, and explore their properties as a function of system parameters. In contrast to the quantum optical case, the condensate dressed states exhibit spatial behaviour which depends on the system parameters, and can be manipulated by changing the cw external field.

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I. INTRODUCTION

Recent experimental work with multiple species Bose Einstein condensates (BEC) [1,2] has motivated theoretical analysis of their wavefunctions [3,4] and excitations [5–7]. The primary tools used to create and investigate these multiple condensates are external microwave and radiofrequency radiation fields, and the combined system is described by a set of coupled Gross Pitaevskii equations (GPE). In quantum optics the eigenstates of the full system of the (single) atom plus field are called *dressed states* (see [8]) and have proved invaluable as calculational and interpretational tools. In this paper we extend this concept to the eigenstates of the coupled GPE, which we shall call *condensate dressed states*.

We consider in detail a condensate with two internal states $|1\rangle$ and $|2\rangle$. We solve for the eigenstates of this system numerically in the case of one spatial dimension, and with plane wave uniform intensity radiation fields. The major new feature that occurs in condensate dressed states is the spatial dependence of the wave functions, which can prove significant even when the external field is a uniform plane wave.

We explore the properties of the condensate dressed states as a function of system parameters, and present results representing broad classes of the possible behaviour. We begin by considering the most general properties of the condensate dressed states including their symmetries. We show that in the simplest case of identical traps for each component and identical collisional interactions, both components have identical spatial behaviour. However, with non identical traps, the two components may have markedly different spatial character, and we examine the dependence of these shapes on the trap parameters, the collisional parameters, and the external field. We show that the condensate's spatial shapes can be manipulated by changing the external field strength or detuning.

II. FORMULATION

The single particle state, $|\Psi\rangle$, of a condensate in a superposition of the two internal states $|1\rangle$ and $|2\rangle$ can be written

$$|\Psi\rangle = \phi_1(\mathbf{r}, t) |1\rangle + \phi_2(\mathbf{r}, t) |2\rangle, \quad (1)$$

where $\phi_i(\mathbf{r}, t)$ is the centre of mass meanfield wavefunction for a particle in state $|i\rangle$. Under the influence of an electromagnetic coupling the wavefunctions associated with each component evolve according to the coupled Gross Pitaevskii equations (GPE)

$$i \frac{\partial \phi_1}{\partial t} = -\nabla^2 \phi_1 + V_1(\mathbf{r})\phi_1 + [w_{11}|\phi_1|^2 + w_{12}|\phi_2|^2] \phi_1 - \frac{\Omega}{2} \phi_2, \quad (2)$$

$$i \frac{\partial \phi_2}{\partial t} = -\nabla^2 \phi_2 + V_2(\mathbf{r})\phi_2 + [w_{22}|\phi_2|^2 + w_{12}|\phi_1|^2] \phi_2 - \frac{\Omega}{2} \phi_1 + \delta_L \phi_2, \quad (4)$$

which describe either a 1-photon [9] or 2-photon Raman [10–12] transition. In Eq. (2) the scaling is chosen as in [9], and Ω and δ_L are the Rabi frequency and the detuning for the transition. The trapping potentials $V_1(\mathbf{r})$ and $V_2(\mathbf{r})$ for atoms in internal states $|1\rangle$ and $|2\rangle$ respectively, may describe any static potential, but here we restrict our attention to the case where they are each harmonic, but possibly with different spring constants and trap centres. The eigenvalue, μ , at $T = 0$ can be identified as the chemical potential of the system. The quantities w_{11} , w_{22} and w_{12} represent the strength of the two intra- and the inter-species interactions, and are proportional to the total number of atoms within the condensed system and the respective scattering lengths. The state $|\Psi\rangle$ is normalised to unity, so that the fractional population n_i of the state $|i\rangle$, is given by the $n_i = \int |\psi_i|^2 d^3r$.

We look for stationary solutions to Eq. (2) of the form $\phi_1(\mathbf{r}, t) = \psi_1(\mathbf{r})e^{-i\mu t}$, $\phi_2(\mathbf{r}, t) = \psi_2(\mathbf{r})e^{-i\mu t}$ which gives

rise to the time independent coupled Gross Pitaevskii Equations

$$\mu \psi_1 = -\nabla^2 \psi_1 + V_1(\mathbf{r})\psi_1 + [w_{11}|\psi_1|^2 + w_{12}|\psi_2|^2] \psi_1 - \frac{\Omega}{2}\psi_2, \quad (5)$$

$$\mu \psi_2 = -\nabla^2 \psi_2 + V_2(\mathbf{r})\psi_2 + [w_{22}|\psi_2|^2 + w_{12}|\psi_1|^2] \psi_2 - \frac{\Omega}{2}\psi_1 + \delta_L \psi_2, \quad (6)$$

In general this equation must be solved numerically, and we have used an optimisation technique (which will be presented elsewhere [13]) to find solutions. We present and discuss

solutions which represent broad classes of possible behaviour, which we discuss in the next two sections.

III. GENERAL PROPERTIES

Many properties of the dressed condensate turn out to be analogous to those of the familiar dressed states of quantum optics [8], however a key difference is the stationary solutions of Eq. (5) are spatially dependent. For simplicity we shall concentrate in this paper on solutions ψ_1 and ψ_2 of Eq. (5), that have no nodes. In this case two eigenstates can be found, which we label $|\Psi_{\pm}\rangle$ and denote the corresponding eigenvalues as μ_{\pm} , where $\mu_+ > \mu_-$. Apart from constant overall phase factors, the mathematical form of these states is

$$\begin{aligned} |\Psi_+\rangle &= \psi_{1+}(\mathbf{r}) |1\rangle - \psi_{2+}(\mathbf{r}) |2\rangle, \\ |\Psi_-\rangle &= \psi_{1-}(\mathbf{r}) |1\rangle + \psi_{2-}(\mathbf{r}) |2\rangle, \end{aligned} \quad (8)$$

where the component wavefunctions $\psi_{1\pm}$ and $\psi_{2\pm}$ are positive real functions. Scanning the detuning of the electromagnetic field from the far red through to the far blue and solving for $|\Psi_{\pm}\rangle$ and μ_{\pm} at each point reveals that the eigenvalues observe an avoided crossing (see Fig. 1(a)), very similar to that seen in quantum optics. Here it is associated with a resonance in which the component populations are near equal (typically when $|\delta_L| < \Omega$), as shown in Fig. 1(b). On either side of this resonance, as $|\delta_L|$ increases, the dressed states approach a single component configuration (i.e the dressed state is almost entirely in one internal state).

The states of Eq. (8) exhibit certain symmetry properties, which relate the solution $|\Psi\rangle = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ for field parameters $\{\Omega, \delta_L\}$ to another solution $|\bar{\Psi}\rangle = \begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \end{pmatrix}$ for $\{\bar{\Omega}, \bar{\delta}_L\}$. For the case of identical trapping potentials ($V_1 = V_2$) and equal collisional strengths

($w_{11} = w_{12} = w_{22}$), the symmetries are the same as those of the quantum optics dressed states, namely if the field is related by the rotation matrix $R(\theta)$ according to

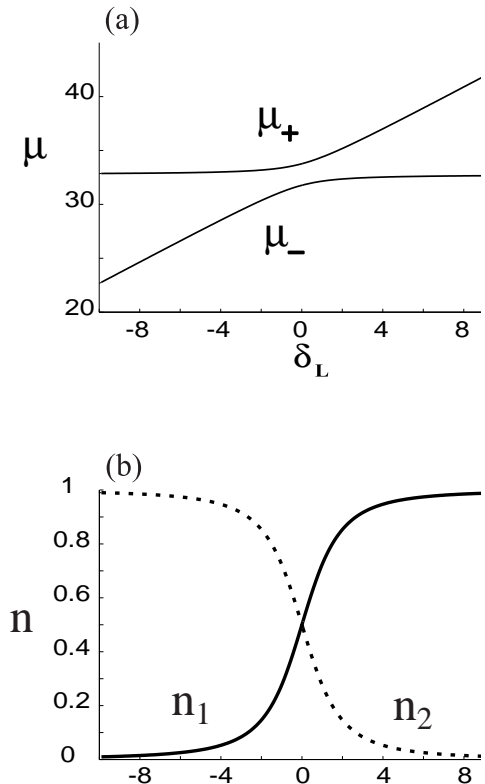


FIG. 1. Properties of condensate dressed states in frequency scans. (a) Eigenvalues μ_+ and μ_- . (b) Populations n_1 (solid line) and n_2 (dashed line) of the internal states $|1\rangle$ and $|2\rangle$. Parameters are $V_1 = V_2 = x^2/4$, $w_{11} = w_{12} = w_{22} = 500$ and $\Omega = 2$.

$$\begin{pmatrix} \bar{\Omega} \\ \bar{\delta}_L \end{pmatrix} = R(\theta) \begin{pmatrix} \Omega \\ \delta_L \end{pmatrix}, \quad (9)$$

where

$$R(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (10)$$

The corresponding dressed eigenvector is given by

$$\begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \end{pmatrix} = R\left(\frac{\theta}{2}\right) \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad (11)$$

with eigenvalue

$$\bar{\mu} = \mu - \delta_L \sin^2 \frac{\theta}{2} - \frac{\Omega}{2} \sin \theta. \quad (12)$$

For $w_{11} = w_{22} \neq w_{12}$ the symmetry is reduced to the single transformation

$$\begin{aligned} \bar{\psi}_1 &= \psi_2, & \bar{\psi}_2 &= \psi_1. \\ \bar{\mu} &= \mu - \delta_L, & \bar{\delta}_L &= -\delta_L. \end{aligned} \quad (13)$$

IV. SPATIAL CHARACTERISTICS OF DRESSED STATES

Perhaps the main interest of the dressed states is that they provide a simple means for manipulating the spatial shape of the condensate components. However this is very dependent on the relative trap potentials and collisional parameters.

In the simplest case, where the trapping potentials are identical ($V_1 = V_2$) and collisional coupling coefficients are all equal, ($w_{ij} = w$), no difference in spatial characteristics of the components is possible. It is easy to show that eigenstates $|\Psi_{\pm}\rangle$ can be written in terms of the eigenfunction ψ_o of the uncoupled one component GPE

$$\mu_o \psi_o(\mathbf{r}) = -\nabla^2 \psi_o + V_1 \psi_o(\mathbf{r}) + w|\psi_o|^2 \psi_o(\mathbf{r}), \quad (14)$$

and take the general form

$$|\Psi_{\pm}\rangle = c_{1\pm} \psi_o(\mathbf{r}) |1\rangle \mp c_{2\pm} \psi_o(\mathbf{r}) |2\rangle, \quad (15)$$

with eigenvalues

$$\mu_{\pm} = \mu_o + \frac{1}{2} \left(\delta_L \pm \sqrt{\delta_L^2 + \Omega^2} \right), \quad (16)$$

where

$$\begin{aligned} c_{1\pm}^2 &= \frac{1}{2} \left(1 \mp \frac{\delta_L}{\sqrt{\delta_L^2 + \Omega^2}} \right), \\ c_{2\pm} &= \mp \sqrt{1 - c_{1\pm}^2}. \end{aligned} \quad (17)$$

These amplitudes ($c_{j\pm}$) and eigenvalues have precisely the same dependence on field parameters Ω and δ_L as for the quantum optics dressed state.

A. Effect of Different Relative Trap Potentials

For simplicity, we will consider only harmonic traps with $V_1 = x^2/4$ and $V_2 = k(x - x_o)^2/4$, which allows condensate $|2\rangle$ to have a different relative spring constant (k) and an offset centre (x_o). This arises in the JILA experiment, for example, because of the different magnetic moment of the two components and the effect of gravity (see [14]). No exact analytic solutions are possible in this case, but the representative behaviour is shown in numerical solutions presented in Figs. 2 and 3. In Fig. 2(a) we see that even a small difference in the relative spring constant (5%) can give rise to a significant difference in the component wavefunctions, and as k is further increased a clear phase separation is observed (Figs. 2(b)-(c)), whereby one species is excluded from the region where the other species has high density.

Offsets between the two trapping potentials also cause spatial reshaping of the two components, as can be seen in Fig. 3, where x_o is successively increased. In Fig. 3(a), where x_o is approximately 1% of the condensate size, significant deformation of component 2 is seen, and when x_o is increased to 1 (i.e 10% of condensate size) phase separation occurs (Figs. 3(b)-(c)).

B. Rabi Frequency Dependence

A scan of the component wavefunction shapes as Ω is varied is shown in Fig. 4, which illustrates the control over the condensate profiles that is afforded by simply altering the strength of the electromagnetic coupling field. Two limiting regimes can be seen. At low fields ($\Omega \ll |\delta_L|$) the components have distinct spatial shapes, determined by different trap potentials and

collisional interactions. On the other hand, in the large field regime ($\Omega \gg |\delta_L|$) Ω becomes the most significant coupling between components and suppresses the spatial differences between the component wavefunctions. In particular, we notice that at large Rabi frequency phase separation between the condensates will be suppressed.

C. Effect of Different Relative Collisional Interactions

All of the previous results have been given for the case of equal collisional interactions ($w_{11} = w_{12} = w_{22}$). Although this is a good approximation to the case of Rubidium, larger variations can be expected for other atomic species, and it may even prove possible to manipulate the relative scattering lengths (e.g see [15]). It has previously been shown that the extent of phase separation of binary condensates in the absence of electromagnetic coupling depends on the relative collisional interactions [3]. Here

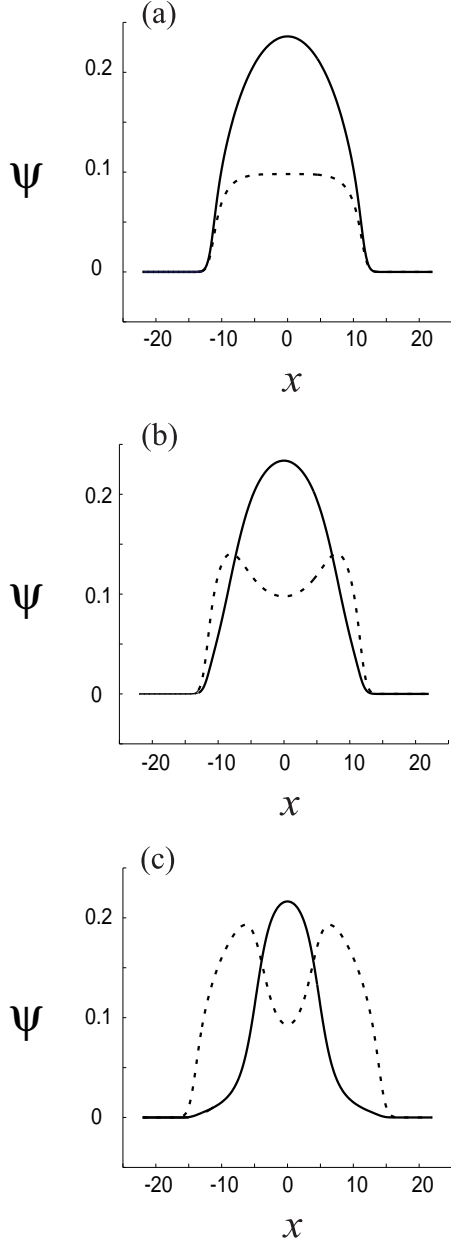


FIG. 2. Effect of relative spring constant on component wavefunctions of dressed states for state $|1\rangle$ (solid line) and state $|2\rangle$ (dashed line). (a) $k = 0.95$, (b) $k = 0.85$, (c) $k = 0.5$. Parameters are as in Fig. 1, except $V_2 = kx^2/4$ and $\delta_L = 2$.

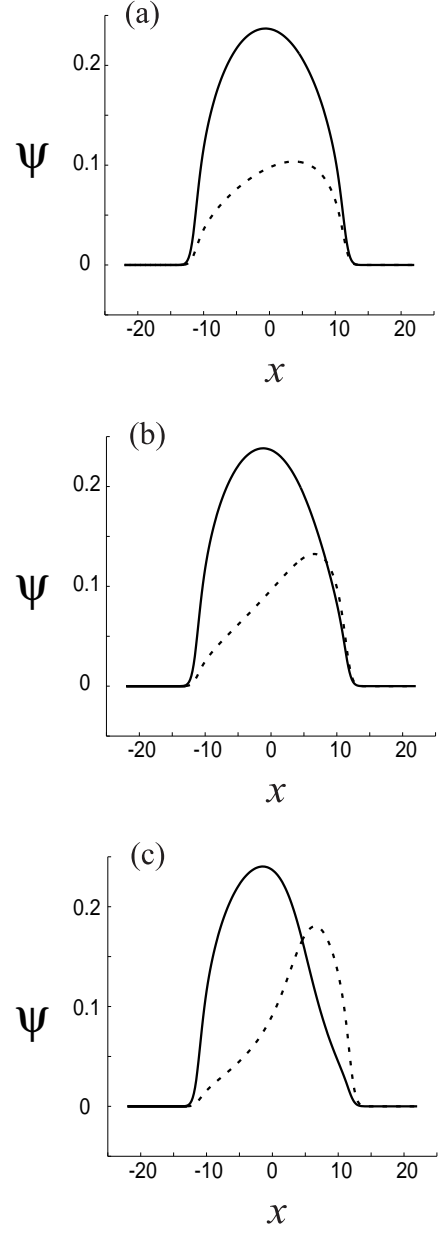


FIG. 3. Effect of trap offset on component wavefunctions. (a) $x_o = 0.2$, (b) $x_o = 0.5$, (c) $x_o = 1$. Parameters are as in Fig. 1, except $V_2 = (x - x_o)^2/4$ and $\delta_L = 2$.

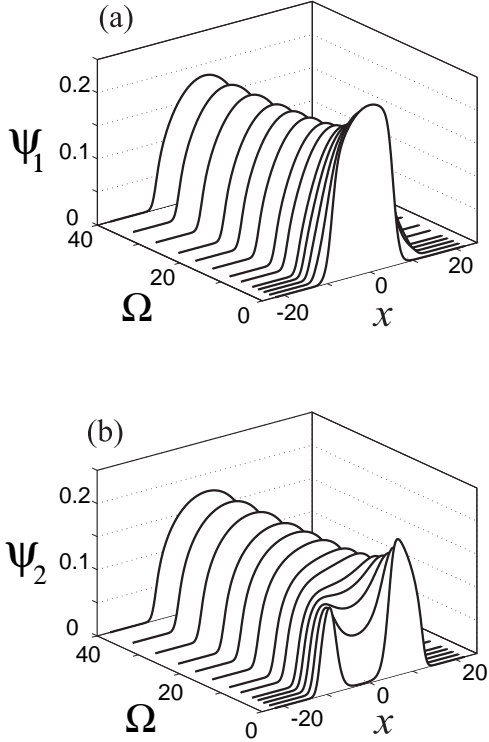


FIG. 4. Effect of changing Rabi frequency on component wavefunctions. (a) component $|1\rangle$. (b) component $|2\rangle$. The trapping potentials are $V_1 = x^2/4$, $V_2 = k(x - x_o)^2/4$ where $k = 0.8$ and $x_o = 0.5$, the detuning is $\delta_L = 2$ and the $w_{ij} = 500$.

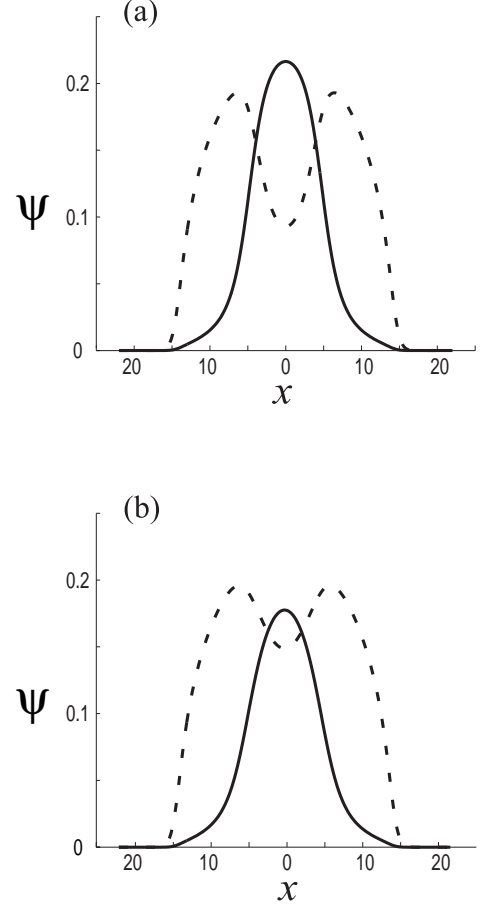


FIG. 5. Effect of changing w_{11} relative to w_{12} and w_{22} on the component wavefunctions, with state $|1\rangle$ (solid line) and state $|2\rangle$ (dashed line). (a) $w_{11} = 500$. (b) $w_{11} = 550$. Other parameters are as in Fig. 2 (c).

we find analogous features arising in our dressed state solutions.

In Fig. 5, we illustrate the effect of increasing w_{11} relative to w_{22} . This has the effect of increasing the self energy of component 1, thus making it less favoured, and resulting in the density of component one being reduced in comparison to component 2. Similarly, decreasing w_{11} makes component 1 energetically favoured, thus causing an increase in its density.

The cross coupling term w_{12} mediates the interaction between the two components. When w_{12} is larger than w_{11} and w_{22} , a competitive interaction occurs, so that the larger component in any region is favoured at the expense of the other component, thereby resulting in density differences being enhanced. Correspondingly, when w_{12} is smaller than w_{11} and w_{22} , it becomes energetically favourable for the components to coexist, and density differences are reduced. The influence that w_{12} exerts on mixing can be clearly shown by examining how the total populations of the two internal states change as the

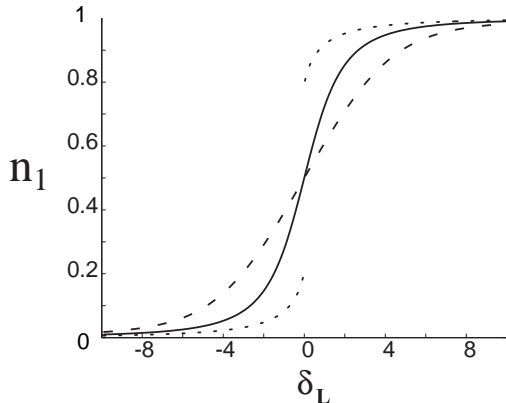


FIG. 6. Component population (n_1) in frequency scans with different interparticle collisional interactions. Solid line $w_{12} = 500$, dashed line $w_{12} = 450$ and dotted line $w_{12} = 550$. Other parameters are $V_1 = V_2 = x^2/4$, $\delta_L = 2$, $\Omega = 2$, and $w_{ii} = 500$.

field frequency is scanned. In Fig. 6 we see that for the case where all collisional rates are equal (solid line) the total population (n_1) in state $|1\rangle$ changes from 0.3 to 0.7 over the range $-1.0 < \delta_L < 1.0$. However, to see the same change in n_1 when w_{12} is decreased by 10% (dotted line), the detuning range has to be extended to $-2.0 < \delta_L < 2.0$. Reducing w_{12} is thus seen to favour mixing of the components. On the other hand, when w_{12} is larger than w_{ii} , mixing is unfavourable and one state will usually dominate the other. This effect can be seen with the dashed line in Fig. 6, where when the frequency scan passes through zero, a very abrupt reversal in the dominant state occurs.

D. Conclusion

We have given a preliminary investigation of condensate dressed states, and explored the similarities and differences to the familiar dressed states of quantum optics. The major new feature that occurs is that the two components of a condensate dressed state may exhibit distinctly different spatial shapes, and we have examined how these shapes depend on the various system parameters. We have considered only the simplest case where the component wavefunctions have no nodes, and the external field is a uniform intensity plane wave, and shown that even in this case the shape of the component wavefunctions can be manipulated by varying the field parameters.

Recent investigations [10,11] have shown that when condensates components are coupled by a spatially varying field, an adiabatic change of detuning may be used to generate an excited condensate state from the ground state. We can generalise our dressed state analysis to include the possibility of a radiation field with spatial

structure, in which case the individual component wave functions may have different numbers of nodes. In this context, the dressed states may be useful to help predict the outcome of adiabatic passage, including the possibility of crossing to a branch of different symmetry.

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